Convex maps, some basic concepts and a new method to generate them

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Introduction

Convex maps were first introduced by Hillier and Hanson in *The social logic of space* (1984), and have since become a standard diagram of space syntax, particularly in the analysis of interior of buildings. Despite their extensive use, computer generation of convex maps is a difficult task. The original guidelines to draw convex maps by hand have been shown to be impossible to translate into a formal description of the type necessary for a computer program (Penn 1997). Algorithmic methods for generating convex maps are useful at a practical level, since a rigorous algorithmic description of the process affords a consistent application of the method and the reproducibility of results. Besides its practical importance, the development of algorithmic methods to generate convex partitions implies also a re-examination of their role as diagrams and representtations of space, and their difference with other types of diagrams used in spatial analysis.

Considering the difficulties of translating into an algorithm the original procedure described by Hillier and Hanson, we have looked at alternatives methods for producing convex maps in the field of shape analysis. In particular we have studied a set of problems which deal with the decomposition of shapes into simpler parts and which are conceptually related to the convex map. Our method uses the medial axis transform, a well known shape descriptor first proposed by H.Blum in 1967, to subdivide architectural plans into non overlapping, convex partitions. Our method produces convincing convex partitions and maps, which often coincide with those resulting from following the original hand-drawn method. Its results can be used for representing the organisation of spaces at the level of simplicity and abstraction of the original convex partitions proposed by Hillier and Hanson, and to allow the study of their configuration through the application of different graph measures and visualisation techniques.

Convex maps

The use of graphs that represent adjacent spaces is central to Space Syntax, patent by their prominence in the works of Hillier, Hanson or Markus (Hillier & Hanson 1984, Hanson 1998, Markus 1993). Originated during the sixties (March and Steadman 1971), these graphs of spatial adjacencies can be seen as the result of applying graph theory to the "bubble diagrams" typical of modernist planning and architecture (Emmons 2006), and which represent relations of adjacency (or permeability, in Space Syntax terminology) between programs, activities or spaces. In our study we are considering convex maps as a subset of this larger family of adjacency or permeability graphs, where the adjacent spatial units are convex areas with certain characteristics, the result of being drawn following a process first outlined by Hillier and Hanson in *the Social Logic of Space*. Initially proposed for the analysis of exterior space (the Y space in alpha-analysis) and later extended to defin-

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ing relevant space partitions in analysis of interior spaces -as for example in " 'Deconstructing' architects' houses" (Hanson 1994)-convex maps afford a geometrical definition for the generation of adjacency graphs that allows social and perceptual theorisation, through concepts such as co-visibility or co-presence.

During the 1980's and 1990's considerable effort was made in the automation of the process originally described, in "the Social Logic of Space" as a set of rules of thumb for drawing convex partitions by hand. This was seen to be of "theoretical as well as practical importance in that it would be hard to maintain that a methodology was objective unless it could be satisfactorily automated" (Penn 1997). In a summary of their findings Penn, Conroy et al. coincide in the complexity of translating the original description of Hiller and Hanson into an algorithm. To overcome some of the limitations, they allowed the possibility of convex spaces to overlap. Much of their effort was focused on using the convex spaces to generate axial maps, and thus this change did not suppose an important difference for their purpose; overlapping convex spaces can also form part of Space Syntax analysis (Hillier 1996), but are the resulting graphs don't capture the adjacency/permeability relations we are interested to represent.

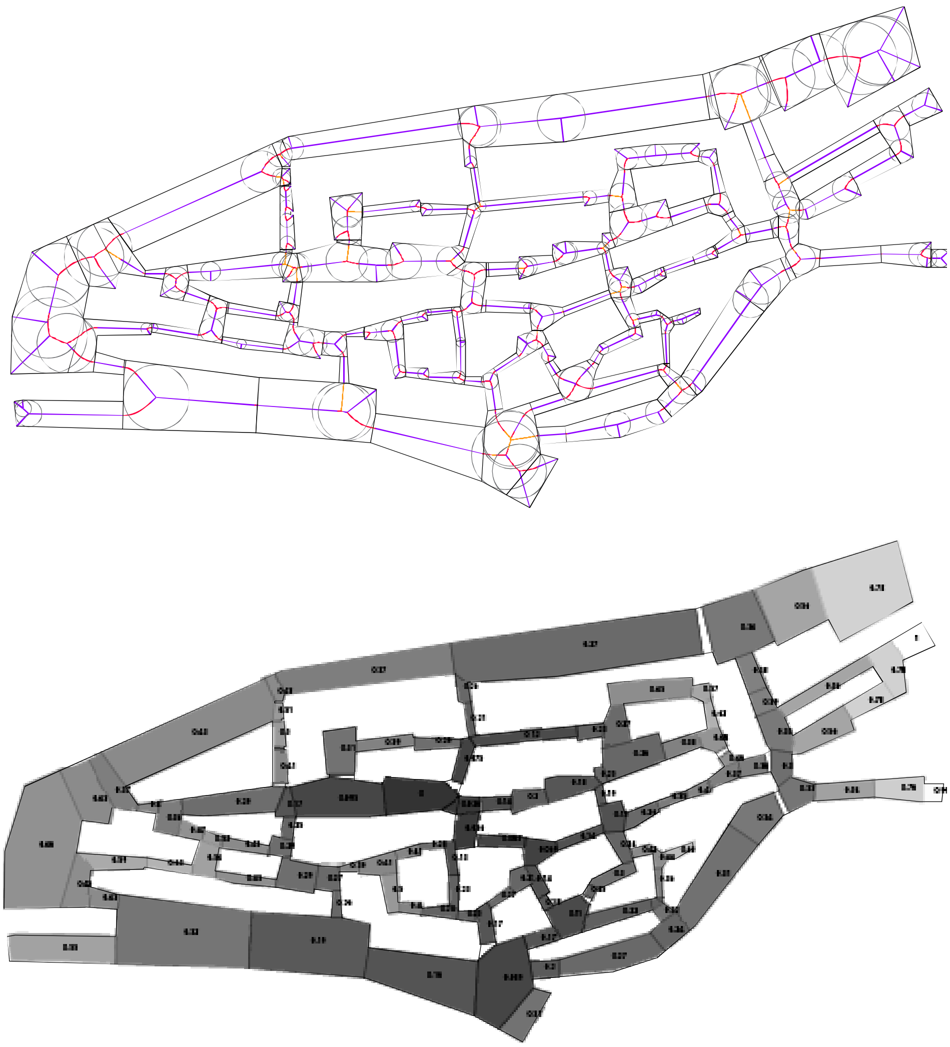


Figure 1: The small village of G, used in *the Social Logic of Space* analysed through our method. On top is the medial axis and all circles centred in branching points (points in the axis with contact with the boundary in more than 2 places). Blue segments show parts of the medial axis produced by 2 straight segments in the boundary, red indicates intervals in the axis generated by the adjacency of a reflex vertex and a straight segment (semi-ligatures), and orange those produced by two reflex points (full-ligatures). The bottom figure shows the result of applying one of our methods (corners first and no concurrency, though the different methods make very little difference in this case), and the calculation of relative asymmetry on the resulting graph. Some small white gaps in the graph belong to small residual areas that have been filtered out. Compar-

ed with the original hand drawn convex map, the differences are very small. In some cases these are produced by imprecisions in the drawing of the original map, other times because some of the aspects of the method, like “fatness” seem to have been ignored in the original drawings in “the Social Logic of Space. It is interesting to see that the degree variation of the connections, at least in this case, is small (most spaces have a degree 2, crossings and small squares, degrees of 3 or 4), particularly compared with the different results in buildings (see figure 3).

Another important contribution to algorithmically generate convex maps are the different partition methods developed by Peponis, Wineman et al. (Peponis et al. 1997). In their work they also emphasise the difficulties of a formal interpretation of the original “paper and pencil” description by Hiller and Hanson. The minimum, e and s partitions they propose subdivide a plan into non-overlapping, convex regions. However, the detail level of these partitions, related to the visibility of different features represented in plan, does not correspond to our intentions either, which is generating simple graphs of adjacent extensions of space in 2 dimensions, capable of describing through simple and sparse graphs permeability, “beadiness” (Hillier & Hanson 1984) and related characteristics of the spatial system.

The scarcity of automatic methods for generating convex maps confirms that the translation of the initial description by Hillier and Hanson into an algorithm is far from straightforward. In our work we have thus used their original heuristic methods as useful guidelines, not as a set of instructions directly translatable into a program. Our goal has been to generate convex maps analogous to the ones proposed by Hiller and Hanson, but not to implement a method that produces identical outcomes. However, we believe that our results are very close to those of the original hand-drawn method, as it is apparent in Figure 1. Our program produces large, non overlapping convex polygons which define the relevant spaces of a building’s layout, and which exhibit a distribution of the degree of their connections when applied to building. This distribution of the degree is an important characteristic of adjacency diagrams, compared to other graph representations of building interiors, such as those produced by treating skeletons directly as graphs (medial axis, straight skeletons or duals of triangulartions, for example), since it captures the different hierarchies resulting from the connections of spaces in a plan.

The medial axis

The field of of shape analysis has produced a number of approaches to the analysis of forms based on shape descriptors, which are close to the intentions and methods of spatial analysis. Shape descriptors are geometrical transformations of a shape that capture features of interest. Some are graph-based, and consider the topological relations of a shape; a category of these is constituted by different types of skeletons, that is, transformations of boundary polygons into graphs that retain some (or all) of the information in the boundary. In our research we have tested a number of skeletons, from simple duals of polygon triangulations to straight skeletons and medial axis transforms. We have finally develop a number of methods for generating convex partitions based on the medial axis, or more accurately, in the geometrically equivalent notion of the segment Voronoi and its dual, the segment Delaunay triangulation (Kirkpatrick 1979).

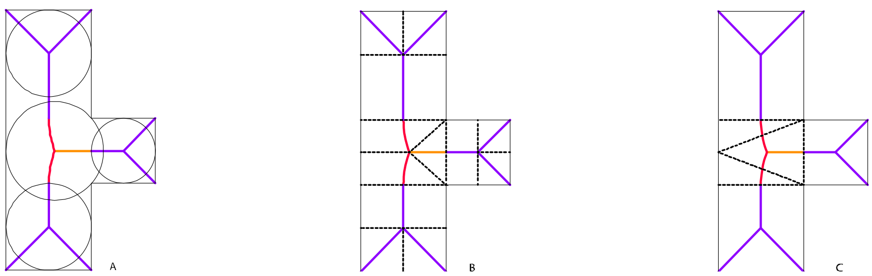
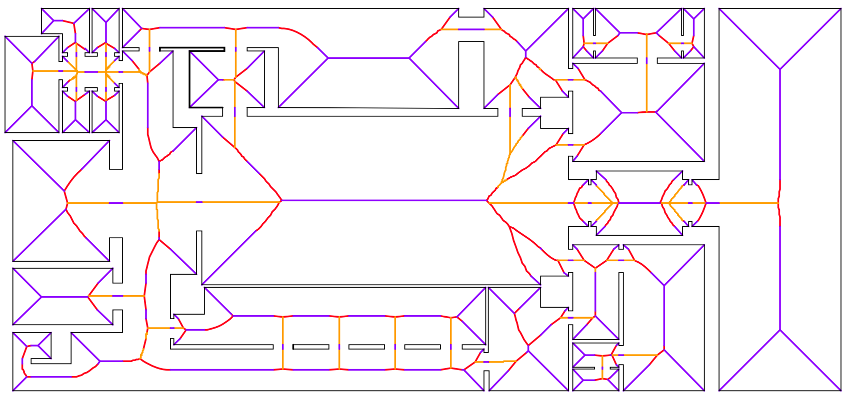


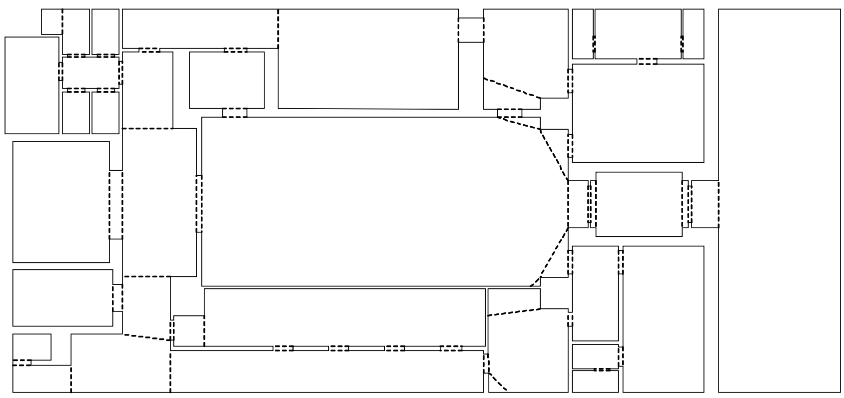
Figure 2: The medial axis of a simple polygon, with straight segments of the axis in blue, semi-ligatures (parabolic sections) in red, and full ligatures in orange. A shows the circles touching to more than 2 boundary points (centred at the branching points). B shows the partition into simplified segments. C shows the partition along symmetric chords of full-ligatures and semi-ligatures, used in our method.

The medial axis transform was first proposed as a form of shape descriptor by Harry Blum (Blum 1967, Blum & Nagel 1978). This work forms the basis for much of subsequent research and analysis methods. Many of these are based on the subdivision of a form into smaller parts that represent its salient features; August, Siddiqi and Zucker proposed for example a shape decomposition technique based on the medial axis (1999) and Tănase and Veltkamp another based on the straight skeleton (2003). Our method is related to these approaches, and implements a form of convex partition based on the medial axis. The resulting subdivisions resemble the outcomes of the paper and pencil method to draw convex maps of Hillier and Hanson (Hillier & Hanson 1984).

The medial axis transform has previously been used to analyse architectural space; its application was initially suggested by March and Steadman in *the Geometry of Environment* (March & Steadman 1971, 194). Other prominent examples can be found in Van Tonder, Lyons et al. analyses of Japanese Zen Gardens (Van Tonder et al. 2002) and Rana and Batty's examples of using shape analysis in architecture (2004). Wen-Chieh Wang (2012) has also proposed the medial axis to generate convex spaces, but these are not strictly convex, their main purpose being the production of axial maps, and not of permeability graphs.

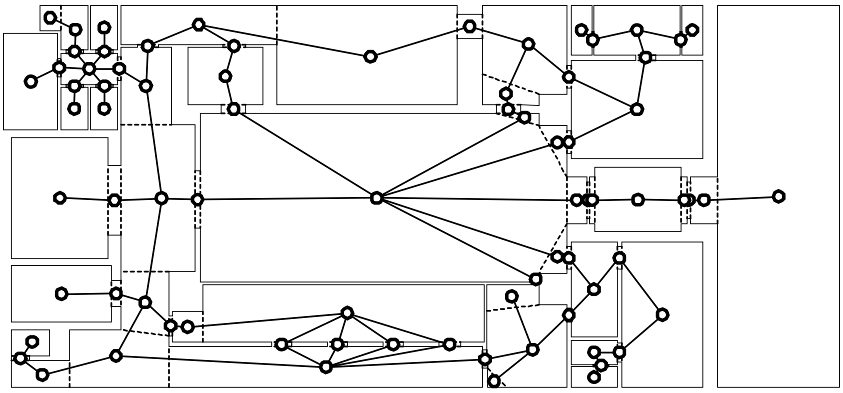


A. Medial axis. Blue indicates that the two boundary sides on the interval are line segments. Red indicates an interval generated by a reflex vertex and a line segment (semi-ligature). Orange, by two points (full-ligature).

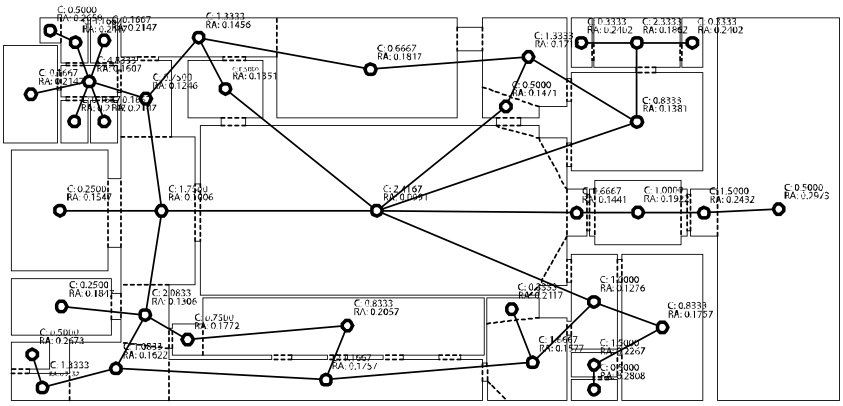


B. Convex subdivision result of the algorithm applied on the medial axis.

Figure 3: The standard method applied to the plan of Luis Sullivan's Peoples Saving Bank, Cedar Rapids, Iowa, 1911. In Figures C and D (below) it is possible to see the effect of the filtering of the graph: figure C shows the graph of the convex map resulting from the strict geometric definitions. Figure D shows the graph after filtering using cues from the medial axis: Doors, small thresholds and residual spaces result of misalignments and small perturbations on the boundary can be identified and dealt with. Observe also the obvious difference in the degree distribution, compared with the analysis in figure 5 of the village of G.



C. Graph of all convex spaces, befo re applying filtering algorithms.



D. Graph (convex map) after filtering th reseholds and co rners, showing cont rol and relative assymetry measu res.

Our method is reminiscent of the decomposition of August, Siddiqi and Zucker (1999), as it also uses Blum and Nagel's ligatures and semi-ligatures as boundaries for significant regions of a shape or boundary. However the need for these regions to be convex in our case, has lead to a different approach, in which rather than decomposing a shape into smaller parts, we additively compose these constituent parts (the final convex partitions) from smaller components. More specifically our method is based on first defining convex sub-regions using the medial axis, and then sequentially adding these subregions into larger convex partitions. There are thus two basic aspects to our method: first, the definition of these initial convex subregions, and second, the rules of addition of these sub-regions into convex partitions.

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